Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Random Graphs. Part 1/Случайные графы. Часть 1
major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 2 (spring) - Grading test

Academic hours: 90 AH in total, including:
lectures: 45 AH.
seminars: 45 AH.
laboratory practical: 0 AH.

Independent work: 45 AH.

In total: 135 AH, credits in total: 3

Author of the program: M.E. Zhukovskiy, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

Annotation

The course is intended for masters of mathematics with an interest in modern discrete mathematics, the probabilistic method, and applications of probability in computer science. The central object studied in the course is a binomial random graph. On his example, the main tools used in the study of random graphs are considered - the method of moments, the method of generating functions, inequalities of deviations of random variables from mathematical expectations such as Yanson's inequality, martingales.

1. Study objective

Purpose of the course

mastering the basic concepts of random graph theory.

Tasks of the course

- students mastering basic knowledge (concepts, concepts, methods and models) in the field of random graphs;
- acquisition of theoretical knowledge and practical skills in the field of random graphs;
- providing advice and assistance to students in conducting their own theoretical research in the field of random graphs.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-2 Able to manage a project through all stages of its life cycle	UC-2.4 Publicly present the project results (or results of its stages) via reports, articles, presentations at scientific conferences, seminars, and similar events
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
Gen.Pro.C-4 Combine and adapt current information and communications technologies (ICTs) to meet professional challenges	Gen.Pro.C-4.3 Create original algorithms and use software tools and modern smart technologies for professional problem-solving
Pro.C-1 Become part of a professional community and conduct local research under scientific guidance using methods specific to a particular professional setting	Pro.C-1.1 Apply principles of scientific work, methods of data collection and analysis, ways of argumentation; prepare scientific reviews, publications, abstracts, and bibliographies on research topics in Russian and English
	Pro.C-1.2 Understand the verification process of software models used to solve related scientific problems

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

fundamental concepts, laws, theories of random graphs;

current problems of the corresponding sections of random graphs;

concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;

basic properties of the corresponding mathematical objects;

analytical and numerical approaches and methods for solving typical applied problems of random graphs.

be able to:

understand the task;
use your knowledge to solve fundamental and applied problems of random graphs;
evaluate the correctness of the problem statements;
strictly prove or disprove the statement;
independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
independently see the consequences of the results;
accurately represent mathematical knowledge in the field of complex computing in oral and written form.

master:

skills of mastering a large amount of information and solving problems (including complex ones);
skills of independent work and mastering new disciplines;
the culture of the formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and random graph methods for their solution;
the subject language of complex calculations and the skills of competent description of problem solving and presentation of the results.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Galton-Watson Branching Processes	8	8		8
2	The law of zero or one for a random graph	7	7		7
3	The method of moments.	8	8		8
4	Models of random graphs.	7	7		7
5	Threshold probabilities	8	8		8
6	Theory of random subsets, binomial and uniform models.	7	7		7
AH in total		45	45		45
Exam preparation		0 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 2 (Spring)

1. Galton-Watson Branching Processes

Central limit theorem for the number of subgraphs of a random graph

2. The law of zero or one for a random graph

The threshold probability theorem for an arbitrary monotone property of random subsets. Determination of the exact threshold probability for a monotonic property, examples.

3. The method of moments.

A sufficient condition for a random variable to be uniquely determined by its moments

4. Models of random graphs.

Classic models: binomial and uniform

5. Threshold probabilities

Possessing monotonic properties by a random subset

6. Theory of random subsets, binomial and uniform models.

Monotonic properties of finite subsets

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Случайные графы [Текст]/В. Ф. Колчин, -М., Физматлит, 2004
2. Модели случайных графов [Текст]/А. М. Райгородский, -М., МЦНМО, 2016

Additional literature

1. Графы. Алгоритмы на языке С [Текст] / В. В. Прут ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т) - М.МФТИ,2017

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 2 (spring) - Grading test

Author: M.E. Zhukovskiy, candidate of physics and mathematical sciences, associate professor, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-2 Able to manage a project through all stages of its life cycle	UC-2.4 Publicly present the project results (or results of its stages) via reports, articles, presentations at scientific conferences, seminars, and similar events
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
Gen.Pro.C-4 Combine and adapt current information and communications technologies (ICTs) to meet professional challenges	Gen.Pro.C-4.3 Create original algorithms and use software tools and modern smart technologies for professional problem-solving
Pro.C-1 Become part of a professional community and conduct local research under scientific guidance using methods specific to a particular professional setting	Pro.C-1.1 Apply principles of scientific work, methods of data collection and analysis, ways of argumentation; prepare scientific reviews, publications, abstracts, and bibliographies on research topics in Russian and English
	Pro.C-1.2 Understand the verification process of software models used to solve related scientific problems

2. Competency assessment indicators

As a result of studying the course the student should:

know:

fundamental concepts, laws, theories of random graphs;
current problems of the corresponding sections of random graphs;
concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;
basic properties of the corresponding mathematical objects;
analytical and numerical approaches and methods for solving typical applied problems of random graphs.

be able to:

understand the task;
use your knowledge to solve fundamental and applied problems of random graphs;
evaluate the correctness of the problem statements;
strictly prove or disprove the statement;
independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
independently see the consequences of the results;
accurately represent mathematical knowledge in the field of complex computing in oral and written form.

master:

skills of mastering a large amount of information and solving problems (including complex ones);
skills of independent work and mastering new disciplines;
the culture of the formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and random graph methods for their solution;
the subject language of complex calculations and the skills of competent description of problem solving and presentation of the results.

3. List of typical control tasks used to evaluate knowledge and skills

Examples of home tasks

1. Consider the binomial subset of $\Gamma = \{1, \ldots, N\}$. Find a threshold probability forcontaining a tuple $x < y < z$ such that $x + y = z$.
2. Let Q be a property of $\Gamma = \{1, \ldots, N(n)\}$, $N(n) \uparrow \infty$, and $p1$, $p2$ be its sharp threshold probabilities. Prove that $p1 \sim p2$, $n \to \infty$.

3. Let ξ have finite $E\xi$ (k) for all $k \in N$. Prove that all $E\xi$ k are also finite and defined uniquely by ($E\xi$ (k), $k \in N$).
4. Let $c \in R$, p = ln n+ln ln n+c+o(1)n Find an asymptotical distribution of the number of vertices having degree 1 in G(n, p).
5. Let L be the set of all first order sentences. Find the set of limits n limn→∞ P G(n, n−2) |= ϕ, $\phi \in$ Lo

.

4. Evaluation criteria

1. Models of random graphs. Classic models: binomial and uniform.
2. Graph random processes. General theory of random subsets, binomial and uniform models.
3. Monotone properties of finite subsets. Examples. The lemma on the monotonicity of the probability of possessing a monotonic property for a random subset. Convex properties, examples.
4. Asymptotic equivalence of the models Γ (p) and Γ (m): the same asymptotic behavior of the probability of possessing a property for random subsets in these models. Two lemmas and a final corollary for monotone properties.
5. Threshold probabilities and threshold functions of possessing monotonic properties by a random subset. The criterion that this function is a threshold probability for the monotonic property Q.
6. The theorem on the existence of a threshold probability for an arbitrary monotonic property of random subsets.
7. Determination of the exact threshold probability for a monotonic property, examples.
8. Small subgraphs in the random graph G (n, p). Function m (G), balanced and strictly balanced graphs, examples. Lemmas on the average number and variance of the number of subgraphs of a random graph G (n, p) isomorphic to a given fixed graph G. A theorem on the threshold probability of the appearance of a subgraph of a random graph G (n, p) isomorphic to a given fixed graph G.
9. The method of moments. A sufficient condition for a random variable to be uniquely determined by its moments. Examples of such random variables. Density and relative compactness of a family of probability measures in metric space. Prokhorov's theorem, a corollary of it. Multidimensional method of moments.
10. Poisson limit theorem for the number of subgraphs of a random graph G (n, p) that are isomorphic to a given fixed strictly balanced graph G. A multidimensional generalization of the Poisson limit theorem. The central limit theorem for the number of subgraphs of a random graph G (n, p) isomorphic to a given fixed graph G.
11. Evolution of a random graph G (n, p). Case np → 0: maximum size and structure of connected components. Limit theorems for the number of components of a fixed size.
12. Evolution of a random graph G (n, p). Case np = $c \in (0, 1)$: theorem on the maximum size of a connected component. Complex and unicyclic components in such a graph are limit theorems for the number of such components. The total number of vertices in
unicyclic components.
13. Evolution of a random graph G (n, p). Case np = c> 1. Branching Galton-Watson processes. The equation for finding the probability of degeneracy. The theorem on the probability of degeneration of a branching process. The size theorem of a maximal connected component of a random graph. Central limit theorem for the size of a maximal connected component.
14. Evolution of a random graph G (n, p). The maximum size of unicyclic and complex components. Asymptotic size order of the maximum tree component of a random graph. The lemma on the absence of complex components of small size. Limited maximum complexity of the component in a random graph. The consequence: the number, size and complexity of complex components.
15. First-order properties in random graphs.
16. The laws of zero or one.

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During examination the student are allowed to use the program of the discipline.